Code No: 181AN **R22**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B. Tech I Year I Semester Examinations, January/February - 2025
MATRICES AND CALCULUS
(Common to CE, ME, ECE, AE, MIE, CSE(AI&ML), CSE(IOT), AI&DS, AI&ML)

Time: 3 Hours **Max. Marks: 60**

Note: This question paper contains two parts A and B.
i) **Part- A** for 10 marks, ii) **Part - B** for 50 marks.

- Part-A is a compulsory question which consists of ten sub-questions from all units carrying equal marks.
- Part-B consists of **ten questions** (numbered from 2 to 11) **carrying 10 marks each**. From each unit, there are two questions and the student should answer one of them. Hence, the student should answer five questions from Part-B.

PART- A

(10 Marks)

1.a) Find the rank of $A = \begin{bmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$. [1]

b) Find the value of k for which the equations have infinite number of solutions
$x + y + z = 1,\ kx - ky + 3z = 5,\ 5x - 3y + kz = 6.$ [1]

c) Write the matrix of the quadratic form $2xy + 6xz - 4yz$. [1]

d) Find the eigen values of A^{-1} where $A = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$. [1]

e) Find 'c' by Rolle's mean value theorem for the function $f(x) = x^2 + 4$ in [-2, 2]. [1]

f) Evaluate $\int_0^\infty x^4 e^{-x} dx$. [1]

g) If $u = \frac{y}{x},\ v = xy$, then find $\frac{\partial(u,v)}{\partial(x,y)}$. [1]

h) Find the stationary point of $f(x, y) = x^2 + 2y^2 - x$. [1]

i) Evaluate $\int_0^1 \int_0^2 y^2 x dy dx$. [1]

j) Change the order of integration for $\int_0^a \int_x^a f(x, y) dy dx$. [1]

PART - B

(50 Marks)

2.a) Solve the system of equations $x + y + z = 6,\ 2x - 3y + 4z = 8,\ x - y + 2z = 5$ by using Gauss - elimination method.

b) Find the values of λ and μ so that the system of equations $2x + 3y + 5z = 9,\ 7x + 3y - 2z = 8,\ 2x + 3y + \lambda z = \mu$ has (i) Unique solution (ii) No solution (iii) Infinite number of solutions. [5+5]

OR

3.a) Find the rank of $A = \begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}$ by reducing it into normal form.

b) Solve the system of equations
$20x + y - 2z = 17,\ 3x + 20y - z = -18,\ 2x - 3y + 20z = 25$ by Gauss Seidel method. [5+5]

4.a) Find the eigen values and the corresponding eigen vectors of the matrix.

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

b) Using Cayley Hamilton theorem, find A^{-1} if $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 5 \\ 0 & -1 & 4 \end{bmatrix}$. [5+5]

OR

5. Reduce the quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy - 2xz + 4zx$ to canonical form by orthogonal transformation and hence find its rank, index, signature, nature. [10]

6.a) Find all values of c by Cauchy's mean value theorem for $f(x) = \frac{x^3 - x^2}{2}$, $g(x) = x^2$ in [0, 3].

b) Expand $\frac{1}{1-x}$ near $x = 0$ using Taylor's series. [5+5]

OR

7.a) Evaluate $\int_0^\infty e^{-x^2}\, dx$ using Gamma function.

b) Find the value of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 1, x = 4$ about the line $y = 1$. [5+5]

8.a) If $u = tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.

b) If $u = x^2 - 2y,\ v = x + y + z,\ w = x - 2y + 3z$, find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. [5+5]

OR

9.a) A rectangular box open at the top is to have volume of 32 cubic ft. Find the dimensions of the box requiring least material for its construction.

b) If $u = \frac{x}{y},\ v = \frac{y}{x}$ verify whether u, v are functionally dependent and if so find the relation between them. [5+5]

10.a) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 dydx$ by changing the order of integration.

b) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$. [5+5]

OR

11.a) Evaluate $\int_0^a \int_y^a \frac{x^2}{\sqrt{x^2+y^2}} dxdy$ by changing into polar coordinates.

b) Using triple integration, find the volume of the sphere $x^2 + y^2 + z^2 = a^2$. [5+5]

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